C2 Jan 2006 (MA)

Q1a) $\underline{f(1)=0}$: $f(1) = 2+1-5+c = 0$

$\therefore c = 5-3$

$\boxed{c = 2}$

b) $f(1)=0$ $\therefore$ $(x-1)$ is a factor of $f(x)$.

$$\begin{array}{r|l} & 2x^2 + 3x - 2 \\ x-1 & 2x^3 + x^2 - 5x + 2 \\ & \underline{2x^3 - 2x^2} \\ & 0 + 3x^2 - 5x \\ & \underline{3x^2 - 3x} \\ & 0 - 2x + 2 \end{array}$$

$\therefore f(x) = (x-1)(2x^2+3x-2)$

but $2x^2 + 3x - 2 = (2x-1)(x+2)$

$\therefore f(x) = (x-1)(2x-1)(x+2)$

c) $2x-3=0$
$2x=3$
$x=\frac{3}{2}$ //

$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}-1\right)(3-1)\left(\frac{3}{2}+2\right) = \boxed{\frac{7}{2}}$ = remainder

Q2a) $(1+px)^9 \simeq (1)^9 + \binom{9}{1}(1)^8(px)^1 + \binom{9}{2}(1)^7(px)^2$

$\simeq \boxed{1 + 9px + 36p^2x^2}$

b) $9p = 36$ and $q = 36p^2$

(By equating coefficients)

$\therefore p = \frac{36}{9} = \boxed{4}$ and $q = 36(4^2) = \boxed{576}$

Q3a) $|AB| = \sqrt{(4-3)^2 + (0-5)^2} = \boxed{\sqrt{26}}$

b) midpoint: $\left(\frac{4+3}{2}, \frac{0+5}{2}\right) \Rightarrow \boxed{\left(\frac{7}{2}, \frac{5}{2}\right)}$

c) midpoint = centre = $\left(\frac{7}{2}, \frac{5}{2}\right)$

AB = diameter = $2 \times$ radius $= \sqrt{26}$

$\therefore r = \frac{1}{2}\sqrt{26}$ so $r^2 = \frac{26}{4} = \frac{13}{2}$

$\Rightarrow \boxed{\left(x - \frac{7}{2}\right)^2 + \left(y - \frac{5}{2}\right)^2 = \frac{13}{2}}$

Q4a) $S_\infty = \dfrac{a}{1-r} = 480$

$\therefore \quad \dfrac{120}{1-r} = 480 \quad \Rightarrow \quad \dfrac{1-r}{120} = \dfrac{1}{480}$

$\overset{\times 120}{\Rightarrow} \quad 1-r = \dfrac{120}{480} = \dfrac{1}{4}$

$\Rightarrow \quad r = 1 - \dfrac{1}{4} = \boxed{\dfrac{3}{4}}$

b) $\left.\begin{array}{l} 5^{th} \text{ term} = ar^4 \\ 6^{th} \text{ term} = ar^5 \end{array}\right\}$ difference $= ar^4 - ar^5$

$= ar^4(1-r)$

$= 120 \times \left(\dfrac{3}{4}\right)^4 \left(1 - \dfrac{3}{4}\right)$

$= \boxed{9.49}$ (2dp)

c) $S_7 = \dfrac{a(1-r^n)}{1-r} = \dfrac{120\left(1 - \left(\frac{3}{4}\right)^7\right)}{1 - \frac{3}{4}} = \boxed{416}$

d) $S_n > 300$

$\dfrac{120\left(1 - \left(\frac{3}{4}\right)^n\right)}{1 - \frac{3}{4}} > 300$

$120\left(1 - \dfrac{3}{4}^n\right) > \dfrac{300}{4}$

$1 - \left(\dfrac{3}{4}\right)^n > \dfrac{300}{4 \times 120}$

$1 - \left(\dfrac{3}{4}\right)^n > \dfrac{5}{8}$

$\therefore \left(\dfrac{3}{4}\right)^n < 1 - \dfrac{5}{8}$

4d) $\left(\frac{3}{4}\right)^n < \frac{3}{8}$

$\log\left(\frac{3}{4}^n\right) < \log\left(\frac{3}{8}\right)$

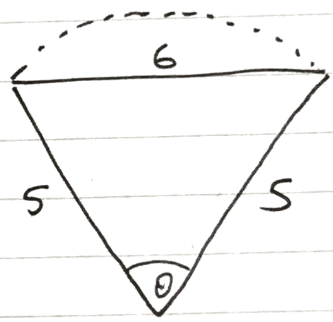
$n\log\left(\frac{3}{4}\right) < \log\left(\frac{3}{8}\right)$

$n > \dfrac{\log\left(\frac{3}{8}\right)}{\log\left(\frac{3}{4}\right)}$ $\therefore n > 3.409..$

signs change when $\div$ by a negative number
$\log\frac{3}{4} < 0$

$\boxed{n_{min} = 4}$

Q5a) cosine rule



$\cos\theta = \dfrac{5^2+5^2-6^2}{2(5)(5)}$

$\cos\theta = \frac{7}{25} = \cos(A\hat{O}B)$

b) $\cos^{-1}\left(\frac{7}{25}\right) = \boxed{1.287^c}$

c) Area $= \frac{1}{2}r^2\theta = \frac{1}{2}\times 5^2 \times 1.287 = \boxed{16.1\ m^2}$

d) Area $\triangle OAB = \frac{1}{2}ab\sin C = \frac{1}{2}\times 5\times 5\times \sin(1.287)$

$= 12m^2$

$\therefore$ Shaded area $= 16.1 - 12 = \boxed{4.1m^2}$

Q6a)

t	15	25	30
V	3.80	9.72	15.37

b) $h = \frac{b-a}{n} = \frac{30-0}{6} = 5$ // [7 values $\therefore n=6$] //

$\therefore$ Area $\simeq \frac{1}{2} \times 5 \left[0 + 15.37 + 2(1.22 + 2.28 + 3.8 + 6.11 + 9.72) \right]$

$\simeq 154.075 \simeq \boxed{154}$

Q7a) $\frac{dy}{dx} = 6x^2 - 10x - 4$

b) $6x^2 - 10x - 4 = 0$

$\Rightarrow 3x^2 - 5x - 2 = 0$

$\Rightarrow (3x+1)(x-2) = 0$

$\therefore 3x + 1 = 0$
$x = -\frac{1}{3}$

$y = 2(-\frac{1}{3})^3 - 5(-\frac{1}{3})^2 - 4(-\frac{1}{3}) + 2$

$= \frac{73}{27}$ //

$x - 2 = 0$
$x = 2$

$y = 2(2)^3 - 5(2)^2 - 4(2) + 2$
$= -10$ //

so $\left(-\frac{1}{3}, \frac{73}{27}\right)$ and $(2, -10)$

are the turning points.

c) $\dfrac{d^2y}{dx^2} = 12x - 10$

d) $\underline{x = 2}$: $\dfrac{d^2y}{dx^2} = 12(2) - 10 = 14 > 0$

$\therefore$ $\underline{(2, -10) \text{ is a minimum point}}$

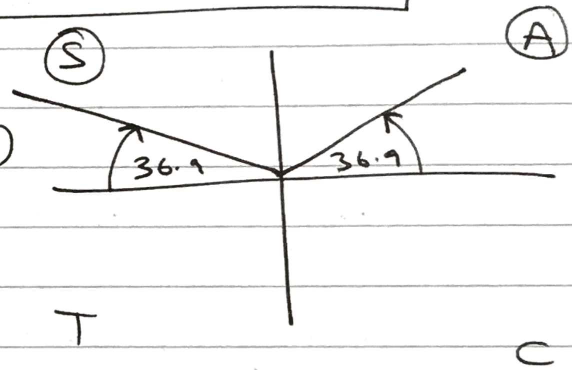
$\underline{x = -\tfrac{1}{3}}$: $\dfrac{d^2y}{dx^2} = 12\left(-\tfrac{1}{3}\right) - 10 = -4 - 10 = -14 < 0$

$\therefore$ $\underline{\left(-\tfrac{1}{3}, \tfrac{73}{27}\right) \text{ is a maximum point.}}$

Q8a) $\sin(\theta + 30°) = \frac{3}{5}$

$\sin^{-1}\left(\frac{3}{5}\right) = \theta + 30° = 36.9°$

Solving in $\boxed{30° \leq \theta + 30° \leq 390°}$



$\theta + 30° = 36.9°, (180 - 36.9°)$

$\theta + 30° = 36.9°, 143.1°$

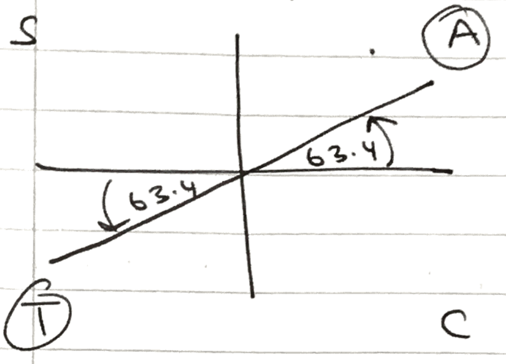
$\boxed{\theta = 6.9°, 113.1°}$

b) $\tan^2\theta = 4 \Rightarrow \tan\theta = \pm\sqrt{4} = \pm 2.$

$\tan\theta = 2$	$\tan\theta = -2$
$\theta = \tan^{-1}(2) = 63.4°$	$\theta = \tan^{-1}(-2) = -63.4°$

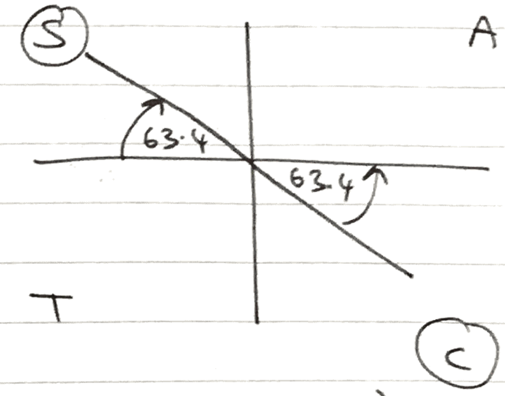
$$\boxed{0 \le \theta < 360}$$

$\theta = 63.4°$,



$$\boxed{\theta = 63.4°, 243.4°}$$

$\theta = -63.4°$



$\theta = (180 - 63.4°), (360 - (63.4°))$

$$\boxed{\theta = 116.6°, 296.6°}$$

Q9 a) $\frac{3}{2} = -2x^2 + 4x$ (equate line and curve)

$2x^2 - 4x + \frac{3}{2} = 0$

$4x^2 - 8x + 3 = 0$

$(2x - 1)(2x - 3) = 0$

$2x - 1 = 0$, $2x - 3 = 0$

$\boxed{x = \frac{1}{2}}$ $\boxed{x = \frac{3}{2}}$

A B

doesn't matter which you take to be y_1 / y_2 as long as you take the absolute value of your final answer

b) $R = \int_{\frac{1}{2}}^{\frac{3}{2}} [y_2 - y_1]\,dx = \int_{\frac{1}{2}}^{\frac{3}{2}} \left[-2x^2 + 4x - \frac{3}{2}\right] dx$

$= \left[-\frac{2x^3}{3} + \frac{4x^2}{2} - \frac{3}{2}x\right]_{\frac{1}{2}}^{\frac{3}{2}} = [0] - \left[-\frac{1}{3}\right] = \boxed{\frac{1}{3}}$